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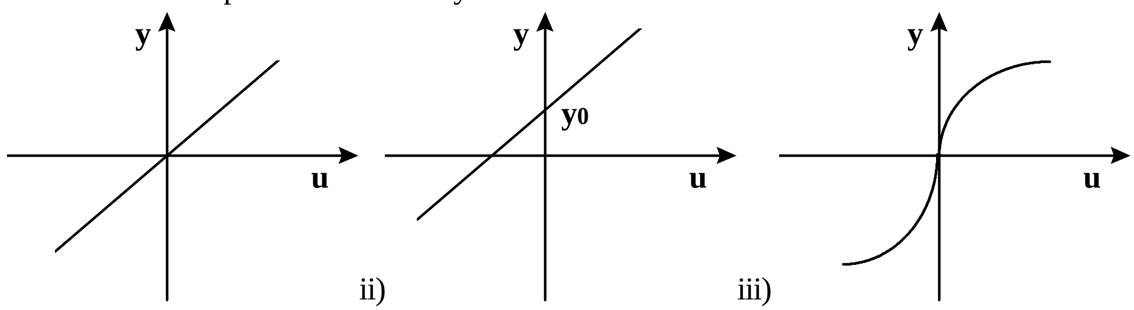
ECEN 5713 System Theory
Spring 1997
Midterm Exam #1

<u>Classification of Systems</u> (20%)

<u>*Problem 1a*)</u> (10%) Consider the memoryless systems given below, in which *u* denotes the input and *y* is the output. Which of them is a linear system ? If not, is it possible to introduce a new output such that the system becomes linear ?

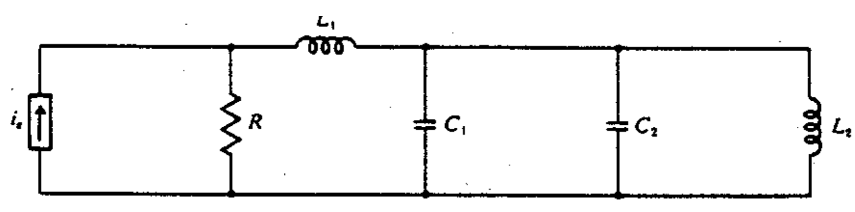


i) ii) iii)

<u>*Problem 1b*)</u> (10%) The impulse response of a linear system is found to be $g(t,\tau)=e^{-|t-\tau|}$ for all t and τ. Is this system causal ? Is it time-invariant ? (Hint: $y(t)=\int_{-\infty}^{\infty} g(t,\tau)u(\tau)d\tau$)

<u>System Representation</u> (20%)

<u>*Problem 2*</u> Find all three representations (i.e., input-output operator, transfer function, and state space equations) of the following RLC circuit,



<u>Linearization</u> (20%)

<u>*Problem 3*</u> A nonlinear system is given by

$$\dot{x}=\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}f_1(x_1,x_2,u)\\ f_2(x_1,x_2,u)\end{bmatrix}=\begin{bmatrix}1+2e^{2x_1}-3(x_2-1)^2+\sin(5u)\\ \frac{1}{3}x_1x_2^3-x_1x_2+2\ln(1+x_1)\end{bmatrix}.$$

Note that $x=\begin{bmatrix}0 & 0\end{bmatrix}^T$ is an equilibrium point at $u=0$. *Linearize* the system about the equilibrium point. To improve the accuracy, approximate up to the *second order* in the linearization process (instead of only the first order: $\nabla_x f$, $\nabla_u f$) in Taylor series expansion. Find the linearized system (my be not in the form of $\{A, B, C, D\}$).

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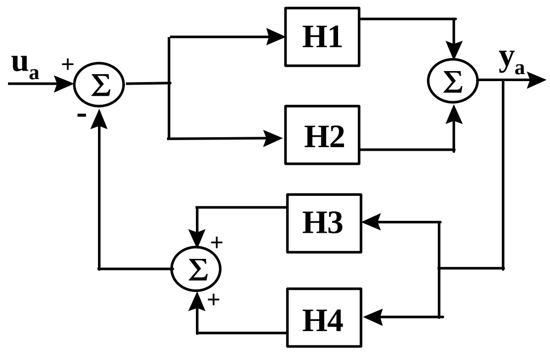
<u>Realization</u> (20%)

<u>Problem 4a</u>) (10%) Find an irreducible (i.e., minimal) controllable canonical form realization (i.e., its simulation diagram and state space equations) for the following system,

$$H(s) = \begin{bmatrix} \dfrac{2s}{s^3 + 6s^2 + 11s + 6} \\ \dfrac{s^2 + 2s + 2}{s^4 + 6s^3 + 9s^2 + 4s} \end{bmatrix} \text{(hint: A is } 6 \times 6\text{).}$$

<u>Problem 4b</u>) (10%) Find the $\{A, B, C, D\}$ matrices of the composite interconnected system given below,



where $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + B\, u_a; \quad y_a = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + D\, u_a$ and

$H_i \equiv \{A_i, B_i, C_i, D_i\}, i = 1,2,3,4$ (hint: you may stop at the temporary variables which are functions of $\{A_i, B_i, C_i, D_i\}$, i = 1, 2, 3, 4).

<u>Linear Algebra</u> (15%)

<u>Problem 5a</u>) (5%) Is it possible to define rules of addition and multiplication such that the set $\{0, 1, 2\}$ forms a field ?

<u>Problem 5b</u>) (5%) The set of all 2×2 matrices with the usual definitions of matrix addition and multiplication is clearly *not* a field. Now consider the set of all 2×2 matrices of the form

$$\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$

where x and y are arbitrary real numbers (i.e., $x, y \in \Re$). Does the set with the usual definitions of matrix addition and multiplication form a field ? If not, show why ? If yes, what are the zero and unity elements ?

Problem 5c) (5%) Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1 \end{bmatrix},$$

find 1) $|A|$, 2) trace(A), 3) $r(A)$, and verify 4)

$$A^{-1} = \begin{bmatrix} -\alpha_{n-1}/\alpha_n & -\alpha_{n-2}/\alpha_n & \cdots & -\alpha_1/\alpha_n & -1/\alpha_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

Evaluation (5%)

Problem 6a) Please rank the difficulty of the problems in Midterm exam #1:
Easiest○ ○ ○ ○ ○Moderate○ ○ ○ ○ ○Hardiest

Problem 6b) Were homework assignments require a reasonable amount of time and effort:
Too simple○ ○ ○ ○ ○Just right○ ○ ○ ○ ○Too demanding

Problem 6c) Were homework assignments:
Too easy ○ ○ ○ ○ ○Just right○ ○ ○ ○ ○ Too difficult

Problem 6d) Describe the pace of the progress:
Too slow○ ○ ○ ○ ○OK○ ○ ○ ○ ○Too fast

Problem 6e) Describe the course content:
Boring○ ○ ○ ○ ○OK○ ○ ○ ○ ○Interesting

Problem 6f) Any other suggestions ? hope ? or wish ?